Review:

Modeling Earthquakes Using Fractal Circular Patch Models with Lessons from the 2011 Tohoku-Oki Earthquake

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Earthquakes occur in a complex hierarchical fault system, meaning that a realistic mechanically-consistent model is required to describe heterogeneity simply and over a wide scale. We developed a simple conceptual mechanical model using fractal circular patches associated with fracture energy on a fault plane. This model explains the complexity and scaling relation in the dynamic rupture process. We also show that such a fractal patch model is useful in simulating long-term seismicity in a hierarchal fault system by using external loading. In these studies, an earthquake of any magnitude appears as a completely random cascade growing from a small patch to larger patches. This model is thus potentially useful as a benchmarking scenario for evaluating probabilistic gain in probabilistic earthquake forecasts. The model is applied to the real case of the 2011 Tohoku-Oki earthquake based on prior information from a seismicity catalog to reproduce the complex rupture process of this very large earthquake and its resulting ground motion. Provided that a high-quality seismicity catalog is available for other regions, similar approach using this conceptual model may provide scenarios for other potential large earthquakes.

Keywords: numerical simulation, dynamic rupture, fractal circular patches, the 2011 Tohoku-Oki earthquake

1. Introduction

Earthquakes occur as dynamic shear rupture process along fault planes that are irregular surfaces or complex networks of a number of surfaces (e.g. [1]) rather than simple mathematical planes. This complexity extends over a wide range that is often assumed to be fractal. The fractal property of fault structures has been studied since the 1980s. Okubo and Aki estimated the fractal dimension of several portions of the San Andreas Fault [2], and Brown and Scholz [3] and Power et al. [4] calculated power spectra for topographic fault surface profiles. Recent developments in surface measurement provide accurate fault geometry power spectra for a broad range (e.g. [5, 6]). These recent studies confirm the idea that earthquake ruptures occur in a fractal fault system. Power spectra calculated from fault traces and fault scarps seem to obey a single power law over nine orders of length scale [6].

Earthquake ruptures are also governed by power laws, as represented by several well-known empirical relations such as Gutenberg-Richter frequency-size statistics (the GR law) and the Omori law for aftershock decay with time. The universality of the GR law has been confirmed both for large earthquakes up to magnitudes over 8 and for very small events observed in very deep mines down to below magnitude -3 [7] and even in very small cracking events in rock experiments (e.g. [8]). Thus, no well-established lower limit is known for the GR law.

Many scaling laws are also provided as power laws. Kanamori and Anderson provided power law relations among macroscopic earthquake parameters such as fault length, fault slip, and seismic moment [9]. They also showed that stress change during an earthquake is a scale-independent parameter, which is to be expected if rupture processes of earthquake are self-similar. Self-similarity is also suggested in the complex rupture propagation of earthquakes. Large earthquakes are characterized by complex rupture processes with power-law statistics (e.g. [10–12]). Complex rupture processes are visible even in very small earthquakes of about magnitude 1 as long as high-frequency components of seismic waves are observed [13].

Despite the fractal nature of earthquakes, studies on quantitative models for earthquake dynamic rupture processes are few. Fukao and Furumoto proposed a conceptual hierarchal model for earthquake rupture growth [14], Seno proposed a fractal asperity model [15], and Otsuki and Ditov proposed a nested structure of fault jogs and segments [16]. These studies demonstrate the importance of the hierarchy in earthquake dynamics and present scaling relations connecting different scales. Using similar ideas of a more quantitative description, we developed a series of earthquake source models using fractal circular patches starting in 2004 [17–21]. In these studies, earthquake dynamic rupture is expressed as a cascading sequence from a small patch to larger patches as illustrated in **Fig. 1**. The present paper first reviews these studies

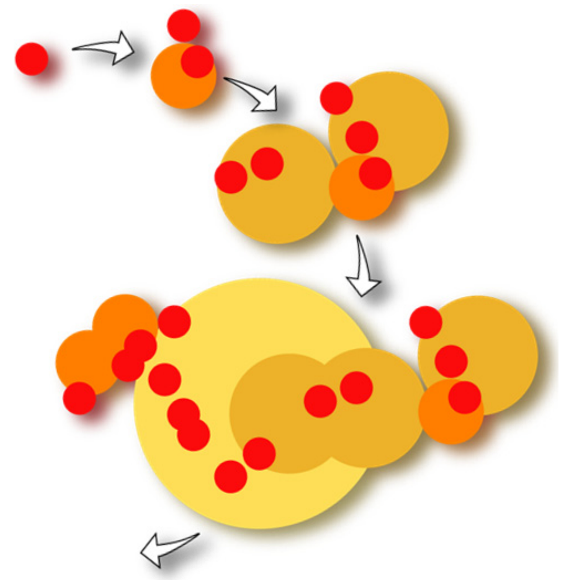

Fig. 1. Schematic illustration of growth process in Fractal circular patch model.

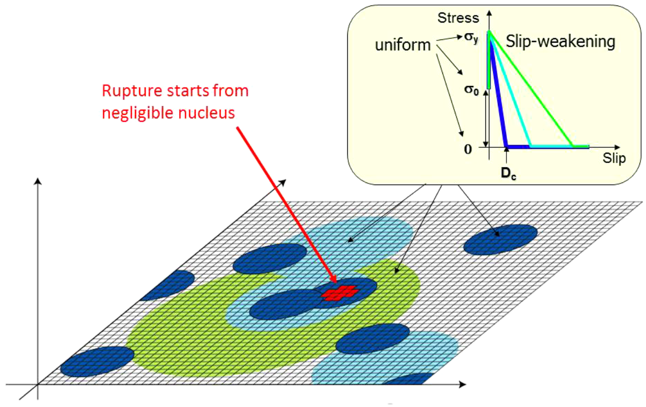

Fig. 2. Example of the patch distribution in IA05 model. Circular patches are distributed on a planer fault. Each patch has slip weakening friction law (inset), with slip weakening distance D_c, which is proportional to the patch radius.

and then discusses perspectives of our model in realistically forecasting earthquake activity.

2. Fractal Circular Patch Model

2.1. Dynamic Rupture Models

Numerical methods widely used to model the dynamic rupture process of earthquakes are finite difference, finite element, and boundary integral methods. The dynamic rupture process is described as a spontaneous propagation of a planer crack with prescribed friction laws in an elastodynamic system. Any numerical method usually discretizes space and time into small units, i.e., finite elements or grids, so any structures smaller than one unit are neglected. Mainly due to limitations on computer memory, the number of finite units for simulating three-dimensional dynamic rupture is restricted and also insufficient for resolving it over a wide range. To overcome this problem, Aochi and Ide ([17], hereafter AI04) developed a renormalized boundary integral method in which the spatiotemporal evolution of the seismic moment rate is conserved between two model spaces having different sizes. This enables a dynamic rupture to be simulated from a very tiny scale to any larger scale (upscaling), although calculation into smaller scales (downscaling) is not taken into account.

It is widely accepted that the dynamic rupture process is governed by a slip weakening law [22] (**Fig. 2**), described by maximum and residual stress levels relative to the initial level, and slip-weakening distance D_c. In AI04, stress levels (maximum σ_y, residual: 0, and initial σ_0) are assumed to be homogeneous and D_c increases with the distance from the hypocenter – an assumption identical to assuming fracture energy linearly increasing from the hypocenter. A dynamic rupture starts from a circular stress drop artificially introduced at a smallest scale, but the effect of this artificial initial process soon disappears and the rupture propagates self-similarly at a constant propagation velocity by renormalizating onto a larger scale. This is consistent with the analytical solution derived by [23], which allowed for infinite stress concentration at the rupture front without any friction law. AI04 confirmed that self-similar rupture propagation is possible under more realistic conditions, i.e., with a friction law and finite stress level.

The size dependence of fracture energy is a requirement coming from fracture mechanics. Slip weakening distance on a laboratory scale of microns [24], while it is an order of meter for the 1995 Kobe earthquake [25]. Estimated fracture energy is 1-10000 J/m^2 for rock experiments (e.g. [1]) and 10^6 J/m^2 for natural earthquakes [26]. Since any earthquake starts from a tiny rupture in our model, we expect an increase in average fracture energy during rupture growth. A large slip may propagate over large-scale heterogeneity, such as jogs, steps, and branching structures, and deformation of these heterogeneities consumes large amounts of energy that is regarded as fracture energy, so it is natural to assign large fracture energy to break large heterogeneous structures. Fracture energy is determined by the product of strength drop and D_c, and the spatial heterogeneity of strength may exist to some extent. A large change in fracture energy of up to six orders is, however, difficult to explain using the spatial difference in strength or stress because stress change during earthquake is almost constant over a wide range (e.g. [9]).

The difference in fracture energy must therefore be explained by scale-dependent change in D_c. This idea is incorporated into a dynamic model by Ide and Aochi ([18] hereafter, IA05) using a fractal circular patch model. This is a simple dynamic rupture model on a flat fault plane embedded in an infinite elastic medium, without any real topography. Instead of real irregularity, this model has irregular fracture energy distribution expressed by randomly distributed circular patches (**Fig. 2**). IA05 assumes seven levels of different-sized patches. Each patch has a slip-weakening friction law with D_c proportional to the patch radius. The radius of $(N-1)$-th level patches are double and the number are a quarter of those of N-th level patches. The area where several patches overlap, D_c is attributed by that of the smallest patch among them. An area without any patch has infinite D_c, which means that slip may occur without stress decrease in this region (background). Like AI04, the stress condition is homogeneous everywhere on the fault plane.

IA05 solves the elastodynamic equation in the same manner as AI04. Initial rupture starting on one of the smallest patches triggers the surrounding patch if the next patch is of similar size to the current rupture size. Such a cascading rupture is the growth mechanism for this patch model. Typically, the rupture size of an N-th level patch breaks surrounding $(N-1)$-th level patches, but is stopped by larger patches and the background. IA05 simulated many events of different size from $M1.3$ to over $M6$ (**Fig. 3**) using renormalization over four nested model spaces. The IA05 model explains the following features of earthquakes:

(1) Frequency-magnitude statistics similar to the GR law.

(2) Uniform stress drop over a wide scale range.

(3) Complex rupture propagation, including subevents and directivity.

(4) Heterogeneous rupture propagation velocity slower than the S-wave velocity on average, but locally accelerated up to P-wave speed, i.e., super-shear-rupture propagation.

(5) Irregular initial phases ahead of main moment release.

Specifically, it is the first time that the last two features are able to be modeled by using large-scale simulation with renormalization. We review these features below.

Rupture propagation velocity in homogeneous media gradually approaches terminal velocity [27]. In the direction of slip, propagation velocity is as fast as P-wave speed. As demonstrated by AI04, the distribution of linearly increasing fracture energy maintains subshear rupture propagation. The fractal patch model has no clear increase in fracture energy from the rupture starting point. Nevertheless, in ruptures cascading from small to large patches, fracture energy averaged over the ruptured area increases quasilinearly, resulting in subshear rupture velocity. The rupture velocity increases within a patch or connected patches where fracture energy is constant. If an area with the same fracture energy extends for a long distance, just by chance, rupture propagation in this area is accelerated to eventually exceed S-wave speed – a feature consistent with the observation of natural earthquakes. Recently, local super-shear-rupture propagation has been reported for shallow inland earthquakes, such as the 2001 Kunlun earthquake [28] and the 2002 Denali

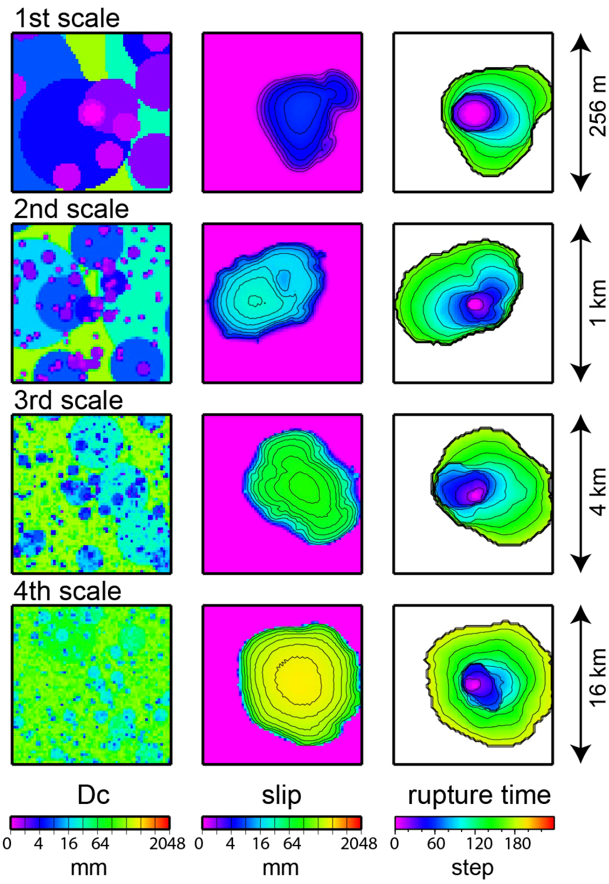

Fig. 3. Example of rupture sequence (after Ide and Aochi, 2005) together with distributions of D_c, slip at the end of calculation, and rupture times on each scale are shown.

earthquake [29], especially along a section in which the fault trace seems relatively simple and straight. In the IA05 concept, we interpret such a simple fault segment as leading to super-shear-rupture propagation as expressed by a single patch or connected patches of the same level.

The initial stage of earthquakes, called the nucleation phase, is another important topic in earthquake seismology, and many studies have tried to identify phases that correlate with final event size [30, 31]. If the very beginning of a seismic waveform carries information about the final event size, this is used to make a rapid estimate of earthquake size for early warning purposes [32]. In most simulated ruptures in IA05, we observe distinct seismic moment pulses considered to be a kind of initial nucleation phase. However, such distinct phases simply reflect rupture progress in small patches preceding breakage in larger patches in a cascading sequence, and the characteristics of waveforms do not differ significantly from that of spontaneously stopping rupture of similar size. Because the probability of cascading can be estimated, we may provide some probabilistic estimates about final size even for such a cascading sequence. However, it does not essentially different from forecasting using the GR law and produces no probabilistic gain.

Yamada and Ide demonstrated that apparent initial phases in IA05 do produce some correlation between parameters in the initial part and final size [33], similar to the previous observation [32]. Nevertheless, discussion remains qualitative for observation and we should not jump to the conclusion that all earthquakes are random cascading sequences. To statistically investigate characteristics of initial seismic waveforms for early warning purposes, a pure cascade model of IA05 would be helpful as a benchmark.

2.2. Long-Term Seismicity Models

One strong hypothesis in IA05 is that every rupture occurs independently in homogeneously stressed media. In nature, the stress state is controlled by long-term plate motion and nearby tectonic events to be very heterogeneous, so considering the effects of stress heterogeneity developing over time, a series of numerical simulations was conducted for given fractal patch models ([19], hereafter, AI09). Five different maps consisting of seven different levels of patches are generated randomly, similar to IA05. Shear stress is slowly increased by tectonic loading, and rupture is initiated at one of the smallest patches at a probability depending on local shear stress level. For a probabilistic distribution function to control initiation, AI09 adopted Weibull distribution, often suggested as strength distribution in rock material and frequently used for probabilistic failure analysis. Once rupture starts, the dynamic rupture process is solved in the same manner as IA05, and stress change due to this event is kept for the following simulation to realize seismic cycles. Complex stress distribution thus develops spontaneously and immediate healing is assumed after each event keeping the same patch distributions in the sequence.

Because small events release only part of the elastic energy loaded by outer tectonic loading, this model eventually produces very large earthquakes that break almost the entire model space. These characteristic earthquakes are repeated at a nearly constant recurrence interval, although, due to randomness introduced into event initiations, we observed certain differences in them. The seismicity pattern between two characteristic earthquakes (an interseismic period) is significantly different. Nevertheless, there are some restrictions on the location of the initial rupture point of characteristic events, and characteristic earthquakes often start from the same area but not exactly the same hypocenters because a rupture in this model is essentially a sequence cascading from small to large patches, and some nested structure of patches, or "a route" developing to the largest patch is necessary to go through the cascade sequence. In other words, some key structure (patch) must be broken to grow into a large event. If such a patch-like nested structure exists in the real fault interface, we could identify the potential hypocenter and routes of future large earthquakes, at least statistically. It is still difficult to identify such characteristic events in nature because none of the real system is ideally isolated and our historical knowledge from instruments is not long enough to observe many seismic cycles. However, our synthetic models may represent some typical feature of observed seismicity. As reported for the Parkfield earthquakes [34], some characteristic earthquakes seem very similar in observed seismograms among 1922, 1934, and 2004, except for one in 1966. Hypocenter locations are actually opposite with respect to the ruptured area for events in 1966 and 2004. This Parkfield example infers an intrinsic fault structure during seismic cycles as simulated in AI09. The probability in similarity and difference is important in seismic hazard assessment.

In the AI09 model, we observe increases of medium to large earthquakes before the characteristic events due to the accumulation of elastic energy stored in model space. However, the AI09 model does not produce active aftershocks, although most numerical simulation occurs commonly without relaxation processes, such as rate-dependent friction or the viscous rheology of surrounding materials.

In the well-known repeating earthquake sequence beneath Kamaishi-city in northeastern Japan, we observe both increases in medium to large earthquakes and few aftershocks. This is a rare example of an almost isolated characteristic event sequence. An explicit patch model similar to the fractal patch model of IA05 was presented with real locations on the plate interface beneath Kamaishi-city [35].

2.3. Lessons from the 2011 Tohoku-Oki Earthquake

Our conceptual fractal patch model, reviewed above, was first applied to the real case of the $M_w 9.0$ 2011 Tohoku-Oki earthquake. The rupture process of this earthquake has been studied by many research groups, and the most specific feature is probably the very large slip that reached the trench that is the main cause of huge tsunamis. In addition to large slip, it has been shown in many papers that the rupture process is quite complex, suggesting the need to introduce a multiscale dynamic model such as IA05.

Ide et al. [36] identified the following four rupture stages consistent with many other slip models:

(1) Relatively slow rupture initiation near the hypocenter.

(2) Downward rupture propagation up to 40 s.

(3) Large slip near the trench at 60-70 s.

(4) Many small ruptures near the bottom edge of the slipped area at about 100 s.

Since the trench located 100 km away from the hypocenter ruptured at 60-70 s after rupture initiation, nominal rupture propagation speed is estimated at about 1.5 km/s. This is significantly slow and requiring an explanation.

Soon after the earthquake, Aochi and Ide ([20], hereafter, AI11) presented a conceptual model explaining the delay in large rupture. This model includes only a few circular patches and a main elongated patch but is sufficient to explain the delay in the large rupture, i.e., by the slow growth of the rupture front due to differences in fracture energy between small and large patches. Different interpretations may explain this delay, for example, with the thermal pressurization process (e.g. [37]). In our terms, such thermal pressurization intervenes in apparent fracture energy.

In the AI11 model, the assumed size and location of patches were not calibrated with any observation. If patch distribution is based on prior information, we would be able to construct a model for forecasting the dynamic rupture process of upcoming earthquakes. This was the motivation of Ide and Aochi ([21], hereafter, IA13) who modeled the 2011 Tohoku-Oki earthquake using more realistic patch distribution. The IA13 model considered four different levels of patches. The location and size of each patch is determined by the earthquake catalog of the Japan Meteorological Agency for about a century, except for an unknown largest patch corresponding to magnitude 9 and one of the smallest patches triggering a medium patch located off the Miyagi coast as shown in **Fig. 4(a)**. After trial-and-error procedures, the typical feature of the Tohoku-Oki earthquake (e.g. [36]) was successfully explained by this simple fractal patch model. In fact, the largest patch in the IA13 model superposes the area where the b-value of the GR law is low. The spatial variation in the b-value is another factor of information that is easy to obtain and that will be helpful in constructing similar realistic models for other regions. We reconsider this possibility in Section 3.

2.4. Strong Motion from Fractal Patch Model

Further interest from the seismic hazard evaluation viewpoint is quantifying the effect of such fractal patch models on ground motion. The most important role is that of heterogeneous patch distribution in different fracture energy controls rupture growth significantly in rupture directivity, rupture velocity, and rupture termination (IA05). The 2011 Tohoku-Oki earthquake is an actual example (AI11, IA13). Based on the change in rupture directivity, we identify different wave fronts (**Fig. 4**) that are typical of this earthquake. Such a complex time series in ground motion is difficult to predict empirically only from kinematic descriptions, but provides dynamic insights that may help in constructing possible ground motion scenarios.

The fractal patch model emphasizes how small patches play some role in initiating an earthquake, (e.g. IA05). However, we have not mentioned enough about their roles during rupture propagation once larger patches are activated. In principle, the difference in fracture energy should lead to certain differences in wave radiation. For the same release of elastic energy, for example, wave radiation is stronger for small fracture energy than for large fracture energy. It is also expected that the former case has higher frequency content because stress is rapidly reduced. However, we must also note that these effects are observable on the unit surface of the fault plane and that the ground motion we observe is the contribution convolved over the entire rupture area. Without very high resolution, it is thus difficult to identify the influence of small patches in radiated ground motion if they are ruptured simultaneously with larger adjacent patches [38].

Another effect of smaller patches on strong motion may be due to irregular geometry of rupture front, as reviewed by [39]. A rupture front progresses faster on smaller patches than on larger adjacent patches (see IA05) because of the difference in fracture energy under a uniform stress condition. This makes the shape of a rupture front complex, and this may affect a radiation pattern in small scale. Note that it is difficult to delay rupture front progress on smaller patches even if we introduce a certain heterogeneity in maximum stress (strength), i.e., smaller patches has larger strength (not larger fracture energy) than larger ones. Initial rupture onset in smaller patches may somehow be delayed (if ruptured, otherwise remains unbroken) due to the heterogeneity, but rupture propagation on them is always fast so that little influence is found when the overall rupture propagation velocity is close to terminal velocity. In this case, ground motion is mainly controlled by the overall rupture process going on in larger patches, and effects of small patches are very limited.

There are several alternatives for smaller patches to influence ground motion during rupture propagation into larger patches. One is a very high stress drop in smaller patches, for which Aochi and Ide demonstrated possibilities [38]. Another is time-dependent frictional processes other than a simple slip-weakening relation. We may also consider off-fault rupture in small patches. There thus remain many subjects to be investigated in ground motion simulation using fractal patch models.

3. Application to Probabilistic Earthquake Forecast

A series of fractal patch models introduced in the previous section suggest the importance of hierarchal structures in seismogenic zones with regard to the understanding of dynamic rupture behavior in large earthquakes. Knowledge of the large structure alone is insufficient for reliably forecasting earthquakes. The Headquarters for Earthquake Research Promotion of Japan has released official long-term forecasts for large earthquakes in subduction zones, dividing them into several independent characteristic units, and neglecting both smaller and larger structures. Neglecting small structures may not cause any practical problems in long-term probability, but not so for large ones.

From the IA13 model viewpoint, the 2011 Tohoku-Oki earthquake was not expected correctly because the largest structure was neglected, although medium and lower-level patches corresponding to the 1978 Miyagi-Oki earthquake were expected. In 1978, a patch was broken without cascading to a larger rupture. In 2011, up

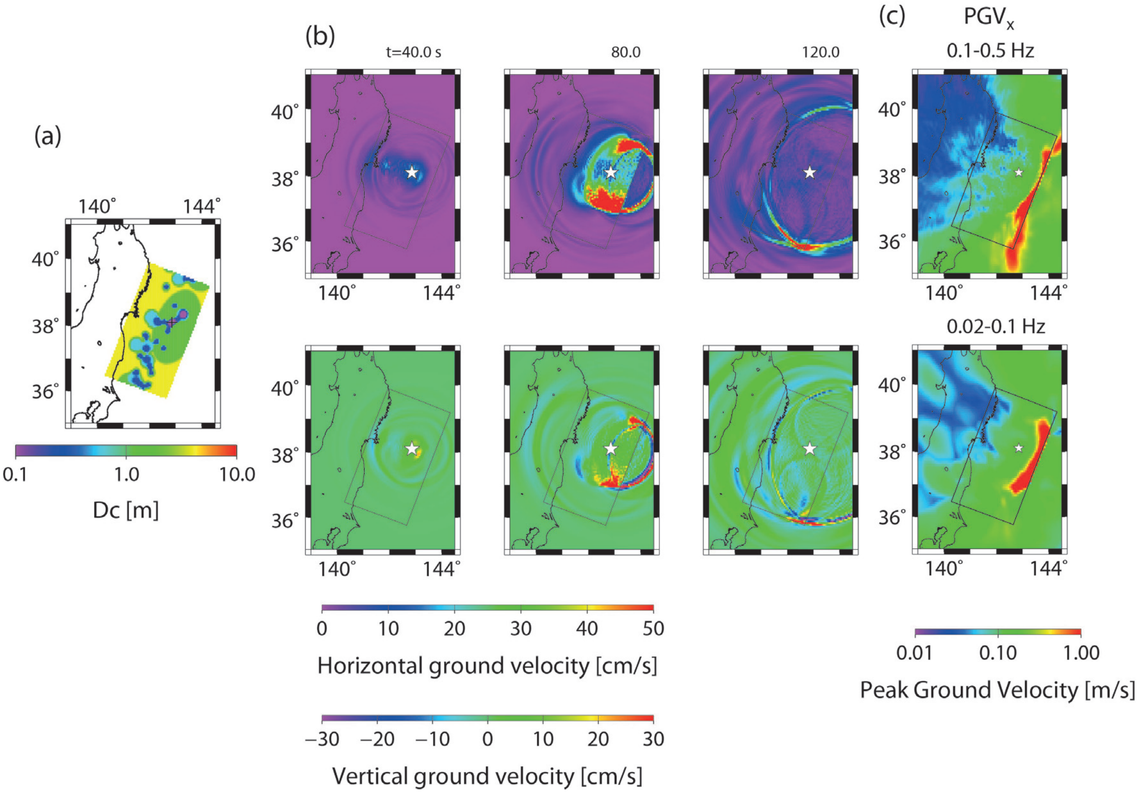

Fig. 4. Ground motion simulation of the 2011 Tohoku-Oki earthquake based on the fractal patch model (after Ide and Aochi, 2013). (a) Assumed patch distribution based on the past seismicity. (b) Ground motion simulation assuming a 1D structure for understanding wave radiation more easily. Top panels show the absolute amplitude of vertical ground motion components and bottom panels show that of vertical ground motion component. (c) Peak ground velocity (PGV) in the east-west (x) component for different frequency ranges.

to the first 40 s, rupture behavior did not differ significantly from a class $M8$ earthquake, which was close to the expected earthquake with 99% in 30 years at that time. However, the rupture of the Miyagi-Oki patch was not the end of the process, but grew to a $M9$ earthquake.

As demonstrated by numerical simulation (e.g. [39] for the 1992 Landers earthquake in complex fault geometry), dynamic rupture propagation is a highly nonlinear process in which even slight differences in stress and friction law may yield quite different results. The difference between the 1978 and 2011 earthquakes in the Tohoku-Oki area is also an example. The dynamic rupture process depends completely on the deterministic physical properties around fault planes and, in principle, a deterministic description of a rupture is possible. However, it is not possible to constrain all of the conditions necessary to make a deterministic prediction of rupture growth starting from a tiny initiation. We may better describe it as a stochastic process [40] with growth probability toward a larger scale. Considering the hierarchal structure of a seismic zone, quantifying such a probability should be an important topic of next-generation earthquake forecasting.

The first step toward such probability forecasting is identifying the hierarchal structure in target regions. The same procedure for IA13 is applicable to other regions. **Fig. 5** shows an example of a patch model constructed for the Hokkaido region northeast of the Tohoku region, compared to b-value distribution and calculated using the

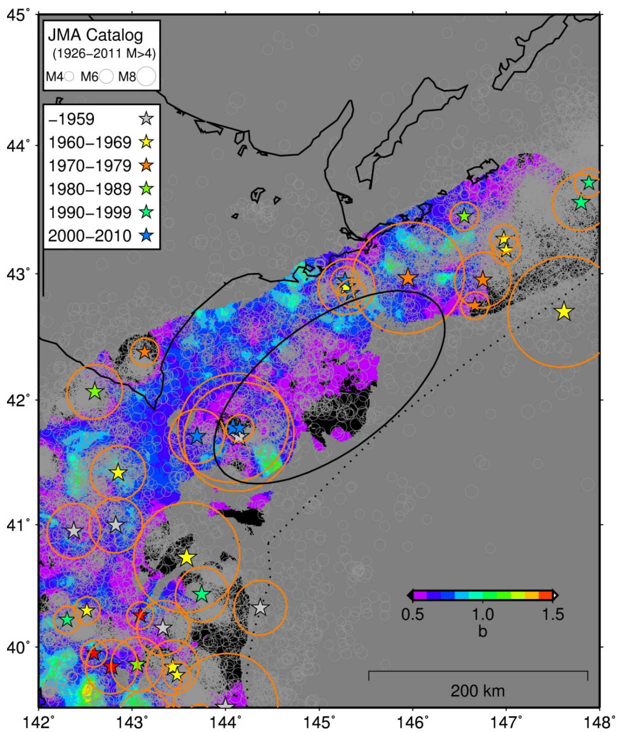

Fig. 5. Example of patch model under the same assumption as for IA13. Colors show b-value distribution and orange circles are patches for three different levels located at the hypocenter of previous earthquakes from 1890 to 2010. The large ellipse is the patch suggested by b-value distribution.

Japan Meteorological Agency seismicity catalog. Earthquakes larger than $M2$ occurring within 10 km vertically from the plate interface were selected from 1990 to the occurrence of the Tohoku-Oki earthquake, similar to IA13. In addition to several medium to large circular patches suggested by previous earthquakes, a very large patch is identified by a large area of very small b-values near the trench axis. Historical earthquakes of about $M9$ have in fact been discovered in this region [41]. This large patch is connected to two $M8$ class patches to the north and to the west. The western patch corresponds to the source for the 1952 and 2003 Tokachi-Oki earthquakes. Like the 1978 Miyagi-Oki earthquake, the previous two earthquakes stopped without triggering the larger patch. However, another similar rupture may trigger a larger patch next time, so a probabilistic forecast must consider the probability of both $M8$-class Tokachi-Oki earthquakes and that of cascading to a larger earthquake of about $M9$.

Once a specific distribution of fractal patches is given, numerical simulation with randomness like AI09 provides possible scenarios of future earthquakes with corresponding probability. Full dynamic modeling is possible, but it remains expensive to consider numerous uncertainties in stress state, friction law, and fault geometry. Alternatively, simple kinematic simulation with some triggering rules may provide quantitatively similar results at a much smaller cost, as shown in IA05. The increase in seismicity of medium to large earthquakes before the largest events observed in the AI09 model may also be used to improve forecasts.

Although the fractal patch model is useful in imagining the source process of earthquakes over a wide range, it remains a very simplified representation of earthquake. Introducing circular patches is an analogy to geometrical irregularities, such as seamounts, horst and graven structures, kinks, and jogs on the plate boundary, but the detailed process are still a question. As demonstrated using 2D numerical simulation [42], we may assign fracture energy distribution based on the geometrical properties of a fractal surface to produce cascading ruptures similar to those of the IA05 model. Nevertheless, for purposes of probabilistic forecasting, the cascading probability between patches is important to be estimated and this may be obtained empirically for individual study regions without knowing further about the physical process in detail. On the other hand, the role of the background remains difficult to quantify. In a series of dynamic rupture simulations, we assumed that the background as the region of infinite slip-weakening distance. This assumption is acceptable for short-term dynamic simulation but obviously insufficient for long-term forecasts. More complex time-dependent friction laws such as rate-and-state dependent friction laws may work better, although they might not be sufficient, either. Further attempts to simulate the behavior of nested circular patches have been attempted [43], which is very expensive but important step for considering the role of background and its effect on the nucleation process. To make better probabilistic forecasts, background should be studied further.

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References:

[1] C. H. Scholz, "The Mechanics of Earthquake and Faulting, 2nd ed.," Cambridge Univ. Press, New York, 2002.

[2] P. G. Okubo and K. Aki, "Fractal geometry in the San Andreas fault system," J. Geophys. Res., Vol.92, pp. 345-355, 1987.

[3] S. R. Brown and C. H. Scholz, "Broad bandwidth study of the topography of natural rock surfaces, " J. Geophys. Res., Vol.90, pp. 12,575-12,582, 1985.

[4] W. L. Power, T. E. Tullis, S. R. Brown, G. N. Boitnott, and C. H. Scholz, "Roughness of natural fault surfaces," Geophys. Res. Lett., Vol.14, pp. 29-32, 1987.

[5] A. Sagy, E. E. Brodsky, and G. J. Axen, "Evolution of fault-surface roughness with slip," Geology, Vol.35, pp. 283-286, 2007.

[6] T. Candela, F. Renard, Y. Klinger, K. Mair, J. Schmittbuhl, and E. E. Brodsky, "Roughness of fault surfaces over nine decades of length scales," J. Geophys. Res., Vol.117, B08409, doi:10.1029/2011JB009041, 2012.

[7] M. Naoi, M. Nakatani, S. Horiuchi, Y. Yabe, J. Philipp, T. Kgarume, G. Morema, S. Khambule, T. Masakale, L. Ribeiro, K. Miyakawa, A. Watanabe, K. Otsuki, H. Moriya, O. Murakami, H. Kawakata, N. Yoshimitsu, A. Ward, R. Durrheim, and H. Ogasawara, "Frequency-magnitude distribution of $-3.7 \leq M_W \leq 1$ mining-induced earthquakes around a mining front and b value invariance with post-blast time," PAGEOPH, doi:10.1007/s00024-013-0721-7, 2013.

[8] C. H. Scholz, "The frequency-magnitude relation of microfracturing in rock and its relation to earthquake," Bull. Seismol. Soc. Am., Vol.58, pp. 399-415, 1968.

[9] H. Kanamori and D. L. Anderson, "Theoretical basis of some empirical relations in seismology," Bull. Seismol. Soc. Am., Vol.65, pp. 1073-1095, 1975.

[10] P. Bernard, A. Herrero, and C. Berge-Thierry, "Modeling directivity of heterogeneous earthquake ruptures," Bull. Seism. Soc. Am., Vol.86, pp. 1149-1160, 1996.

[11] M. Mai and G. C. Beroza, "Source Scaling Properties from Finite-Fault-Rupture Models," Bull. Seismol. Soc. Am., Vol.90, pp. 604-615, 2000.

[12] S. Ide, "Scaling relations for earthquake source process," Zishin 2, Vol.61, pp. S329-S338, 2009.

[13] T. Yamada, J. J. Mori, S. Ide, H. Kawakata, Y. Iio, and H. Ogasawara, "Radiation efficiency and apparent stress of small earthquakes in a South African gold mine," J. Geophys. Res., Vol.110, B01305, 2005.

[14] Y. Fukao and M. Furumoto, "Hierarchy in earthquake distribution," Phys. Earth Planet. Inter., Vol.37, pp. 149-168, 1985.

[15] T. Seno, "Fractal asperities, invasion of barriers, and interplate earthquakes," Earth Planets Space, Vol.55, pp. 649-665, 2003.

[16] K. Otsuki and T. Dilov, "Evolution of hierarchical self-similar geometry of experimental fault zones: Implications for seismic nucleation and earthquake size," J. Geophys. Res., Vol.110, B03303, doi:10.1029/2004JB003359, 2005.

[17] H. Aochi and S. Ide, "Numerical study on multi-scaling earthquake rupture," Geophys. Res. Lett., Vol.31, L02606, doi:10.1029/2003GL018708, 2004.

[18] S. Ide and H. Aochi, "Earthquakes as multiscale dynamic rupture with heterogeneous fracture surface energy," J. Geophys. Res., Vol.110, B11303, doi:10.1029/2004JB003591, 2005.

[19] H. Aochi and S. Ide, "Complexity in earthquake sequences controlled by multiscale heterogeneity in fault fracture energy," J. Geophys. Res., Vol.114, B03305, doi:10.1029/2008JB006034, 2009.

[20] H. Aochi and S. Ide, "Conceptual multi-scale dynamic rupture model for the 2011 Tohoku earthquake," Earth Planets and Space, Vol.63, pp. 761-765, doi:10.5047/eps.2011.05.008, 2011.

[21] S. Ide and H. Aochi, "Historical seismicity and dynamic rupture process of the 2011 Tohoku-Oki earthquake," Tectonophysics, Vol.600, pp. 1-13, doi:10.1016/j.tecto.2012.10.018, 2013.

[22] Y. Ida, "Cohesive force across the tip of a longitudinal-shear crack and Griffith's specific surface energy," J. Geophys. Res., Vol.77, pp. 3796-3805, doi:10.1029/JB077i020p03796, 1972.

[23] B. V. Kostrov, "Selfsimilar problems of propagation of shear cracks," PMM, Vol.28, pp. 889-898, 1964.

[24] M. Ohnaka, "A constitutive scaling law and a unified comprehension for frictional slip failure, shear fracture of intact rock, and earthquake rupture," J. Geophys. Res., Vol.108, 2080, doi:10.1029/2000JB000123, 2003.

[25] S. Ide and M. Takeo, "Determination of constitutive relations of fault slip based on seismic wave analysis," J. Geophys. Res., Vol.102, pp. 27379-27391, 1997.

[26] G. C. Beroza and P. Spudich, "Linearized inversion for fault rupture behavior: Application to the 1984 Morgan Hill, California, earthquake," J. Geophys. Res., Vol.93, pp. 6275-6296, 1988,

[27] D. J. Andrews, "Rupture velocity of plane strain shear cracks," J. Geophys. Res., 81, pp. 5679-5687, 1976.

[28] M. Bouchon and M. Vallée, "Observation of long supershear rupture during the $M = 8.1$ Kunlunshan earthquake," Science, Vol.301, pp. 824-826, 2003.

[29] E. M. Dunham and R. J. Archuleta, "Evidence for a Supershear Transient during the 2002 Denali Fault Earthquake," Bull. Seismol. Soc. Am., Vol.94. pp. S256-S268, 2004.

[30] Y. Iio, "Observations of the slow initial phase generated by microearthquakes: Implications for earthquake nucleation and propagation," J. Geophys. Res., Vol.100, pp. 15333-15349, doi:10.1029/95JB01150, 1995.

[31] W. L. Ellsworth and G. C. Beroza, "Seismic evidence for an earthquake nucleation phase," Science, Vol.268, pp. 851-855, 1995.

[32] E. L. Olson and R. M. Allen, "The deterministic nature of earthquake rupture," Nature, Vol.438, pp. 212-215, doi:10.1038/nature04214, 2005.

[33] T. Yamada and S. Ide, "Limitation of the Predominant-Period Estimator for Earthquake Early Warning and the Initial Rupture of Earthquakes," Bull. Seismol. Soc. Am., Vol.98, pp. 2739-2745, 2008.

[34] W. H. Bakun, B. Aagaard, B. Dost, W. L. Ellsworth, J. L. Hardebeck, R. A. Harris, C. Ji, M. J. Johnston, J. Langbein, J. J. Lienkaemper, A. J. Michael, J. R. Murray, R. M. Nadeau, P. A. Reasenberg, M. S. Reichle, E. A. Roeloffs, A. Shakal, R. W. Simpson, and F. Waldhauser, "Implications for prediction and hazard assessment from the 2004 Parkfield earthquake," Nature, Vol.437, 9690974, doi:10.1038/nature04067, 2005.

[35] N. Uchida, T. Matsuzawa, W. L. Ellsworth, K. Imanishi, T. Okada, and A. Hasegawa, "Source parameters of a M4.8 and its accompanying repeating earthquakes off Kamaishi, NE Japan: Implications for the hierarchical structure of asperities and earthquake cycle," Geophys. Res. Lett., Vol.34, L20313, doi:10.1029/2007GL031263, 2007.

[36] S. Ide, A. Baltay, and G. C. Beroza, "Shallow dynamic overshoot and energetic deep rupture in the 2011 M_w9.0 Tohoku-Oki earthquake," Science, Vol.332, pp. 1426-1429, doi:10.1126/science.1207020, 2011.

[37] Y. Mitsui, Y. Iio, and Y. Fukahata, "A scenario for the generation process of the 2011 Tohoku earthquake based on dynamic rupture simulation: role of stress concentration and thermal fluid," Earth Planets Space, Vol.64, pp. 1177-1187, 2012.

[38] H. Aochi and S. Ide, "Ground motions characterized by a multi-scale heterogeneous earthquake model," Earth Planets and Space, 2014 (in press).

[39] A. A. Gusev, "High-frequency radiation from an earthquake fault: A review and a hypothesis of fractal rupture front geometry," PAGEOPH, Vol.170, pp. 65-93, 2013.

[40] H. Aochi , E. Fukuyama, and R. Madariaga, "Constraints of Fault Constitutive Parameters Inferred from Non-planar Fault Modeling," Geichemistry, Geophysics, Geosystems, Vol.4, No.2, doi:10.1029/2001GC000207, 2003.

[41] H. Aochi, M. Cushing, O. Scotti, and C. Berge-Thierry, "Estimating rupture scenario likelihood based on dynamic rupture simulations: the example of the segmented Middle Durance fault, southeastern France," Geophys. J. Int., Vol.165, pp. 436-446, doi:10.1111/j.1365-246X.2006.0284.x, 2006.

[42] F. Nanayama, K. Satake, R. Furukawa, K. Shimokawa, B. F. Atwater, K. Shigeno, and S. Yamaki, "Unusually large earthquakes inferred from tsunami deposits along the Kuril trench," Nature, Vol.424, pp. 660-663, 2003.

[43] S. Ide, "Dynamic rupture propagation on a 2D fault with fractal frictional properties," Earth Planets Space, Vol.59, pp. 1099-1109, 2007.

[44] H. Noda, M. Nakatani, and T. Hori, "Large nucleation before large earthquakes is sometimes skipped due to cascade-up – Implications from a rate and state simulation of faults with hierarchical asperities," J. Geophys. Res. Solid Earth, Vol.118, pp. 2924-2952, doi:10.1002/jgrb.50211, 2013.

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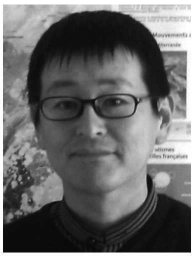

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